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### Thermoelastic Properties and Specific Heat Curve of C60 in Fcc Phase

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#### Abstracts

We have applied an Extended Three Body Force Shell Model (ETSM) for the investigation of thermoelastic properties such as cohesive energy ( $\phi$ ), molecular force constant ( $f$ ), compressibility ( $\beta$ ), Restrahlen frequency ( $\omega$ ), Debye temperature ( $\theta_D$ ), Gruneisen parameter ( $\gamma$ ), second Gruneisen parameter ( $q$ ), Moelwyn Hughes constants ( $F_1$ ), and ratio of volume expansion coefficient ( $\alpha$ ) to volume specific heat ( $cv$ ) of C60 in Fcc phase. We have also reported the TOECs, pressure and temperature derivatives of SOECs. Here, we have measured the specific heat curve of C60. Here, we could compare our result with experimental data of neutron model and microcalorimeter data. Our computed specific heat results follow the same trends of variation with corresponding experimental data. The values of specific heats also increase with temperature. Our calculated results on specific heat are in closer agreement with the experimental data. Here, we could compare our result with other available data for some properties only at room temperature.

**Keywords:** Elastic Constants, Cohesive energy, Specific Heat.

#### Introduction

In the present decade, considerable interest has been taken in the investigation of anharmonic properties of materials of various kinds[1-2]. Many workers have contributed in the field through their experimental and theoretical approach. Several efforts have been made in the study of physical and anharmonic properties of solids of different types[3-6]. Utilizing different physical conditions and using several techniques. Some interesting results have been presented by several investigators while studying the anharmonic properties of the substances possessing various crystal structures. Some have studied temperature variation of anharmonic properties of mixed alkali halides and cyanides of rare gas materials[7]. of alkali halides[8-10]using ultrasonic[11-12]and Brillouin scattering[13] methods.

The static and dynamic structural properties of solid C<sub>60</sub> are currently the subject of numerous investigations. At room temperature, the crystal has a face-centered cubic fcc structure, the fcc phase has the symmetry of the space group Fm3m with a lattice constant of  $a_0=14.2\text{\AA}$ , where the C<sub>60</sub> molecules rotate freely. With decreasing temperature, the rotational degrees of freedom of the C<sub>60</sub> molecules becomes more and more hindered and the solid C<sub>60</sub> undergoes a first order structural phase transition from orientationally disordered fcc phase to an orientationally ordered simple cubic structures with four molecules per unit cell located on four different cubic sub lattices having different orientations[14].

We have developed an Extended Three-Body Force Shell Model (ETSM), this model has successfully been applied to study the static, dynamic, dielectric, optical and anharmonic properties of pure ionic crystals, cyanides and alkali halide- cyanide mixed crystals having NaCl and CsCl structures. Looking to the considerable success of the ETSM[15-24], we have applied the ETSM to calculate the cohesive, thermal and elastic properties of C<sub>60</sub>.

The essential formulation of the present model is given in the next section and input data, model parameters, results discussion and conclusions obtained by us are presented and discussed in the subsequent section.

#### Formulation

The interaction potential used to derive the framework of the present ETSM can be expressed as [25]

$$\phi = -\frac{e^2}{2} \sum_{kk'} Z_k Z_{k'} r_{kk'}^{-1} \left[ 1 + \sum_{kk''} f_k(r_{kk''}) \right] - \sum_{kk'} c_{kk'} r_{kk'}^{-6} - \sum_{kk'} d_{kk'} r_{kk'}^{-8} - b \sum_{kk'} \beta_{kk'} \exp \left\{ \frac{r_k + r_{k'} - r_{kk'}}{\rho} \right\} \quad (1)$$

where the first two terms represent the long range Coloumb and Three Body Interaction [26] the third and fourth terms are the van der Waals (vdW) interaction due to dipole-dipole (d-d) and dipole-quadrupole (d-q) attractions, the fifth term represents

the Hafemeister Flygare[27] type short- range overlap repulsion extended up to second neighbour ions,  $\beta_{kk'}$  are the Pauling coefficient[28] given by

$$\beta_{kk'} = 1 + \left( \frac{z_k}{n_k} \right) + \left( \frac{z_{k'}}{n_{k'}} \right) \quad (2)$$

with  $z_k$  ( $z_{k'}$ ) and  $n_k$  ( $n_{k'}$ ) as the valency and the number of effective electrons in the outermost orbit of the cations (anions).

The range and hardness ( $b$  and  $\rho$ ) parameters determined from the equilibrium condition

$$[d\phi(r)/dr]_{r=r_0}=0 \quad (3)$$

Where  $r$  is the nearest interionic separation and  $r_0$  is the equilibrium separation and the bulk modulus,

$$B=(9Kr_0)^{-1}[d^2\phi(r)/dr^2]_{r=r_0} \quad (4)$$

With  $K$  as the crystal structure constant ( $K=2$  for NaCl structure).

**a. Cohesive and Thermal properties:** Cohesive energy ( $\phi$ ) of a material is a fundamental property, which tells us about the cohesion in the system. If the cohesion for the material is negative it means that the constituent atoms will gain energy by reacting to form a compound (solid or molecule). The binding energy in ionic crystals can be explained by calculating their cohesive energy from large number of interatomic interaction potentials. The details of these potentials have been reviewed earlier by [29-30]. The other parameters like molecular force constant ( $f$ ), compressibility ( $\beta$ ), Restrahlen frequency ( $\nu_0$ ), Debye temperature ( $\theta_D$ ), Gruneisen parameter ( $\gamma$ ), Moelwyn Hughes constants ( $F_1$ ) and ratio of volume thermal expansion coefficient ( $\alpha_v$ ) to volume specific heat ( $C_v$ ) which are directly derived from the cohesive energy  $\phi(r)$  are computed on the lines of Singh and Gaur [25].

$$\beta = \frac{3Kr_0}{f} \quad (5)$$

Where,  $K$  is the crystal structure constant ( $K=1.54$  structure),  $r$  is the interatomic separation and  $f$  is the molecular force constant given as,

$$f = \frac{1}{3} \left[ \phi''_{kk'}(r) + \frac{2}{r} \phi'_{kk'}(r) \right]_{r=r_0} \quad (6)$$

Where,  $\phi''(r)$  and  $\phi'(r)$  are second and first order derivatives of potential energy. The force constant, in turn, gives the Restrahlen frequency as,

$$\nu_0 = \frac{1}{2\pi} \left( \frac{f}{\mu} \right)^{1/2} \quad (7)$$

Where,  $\mu$  is the reduced mass of the system.

In addition to cohesive properties, we have calculated the thermal properties for the  $C_{60}$  for the room

temperature. In order to complete the test of any model, one has to apply it for the study of various thermal properties. Our model not only provides information about the temperature variation of thermal properties but also provide much insight into the intermolecular forces in solids. In this context, we have computed the Debye temperature ( $\theta_D$ ), Gruneisen parameter ( $\gamma$ ), second Gruneisen parameter ( $q$ ), ratio of volume expansion coefficient ( $\alpha_v$ ) to volume specific heat ( $c_v$ ) and the Moelwyn Hughes constants ( $F_1$ ).

The infrared absorption frequency (Reststrahlen frequency)  $\nu_0$  gives us the Debye temperature as,

$$\theta_D = \frac{h\nu_0}{k} \quad (8)$$

Here  $h$  and  $k_B$  are Planck's and Boltzmann constant respectively. The Gruneisen parameter is given as,

$$\gamma = -\frac{r_0}{6} \left[ \frac{\phi'''(r)}{\phi''(r)} \right]_{r=r_0} \quad (9)$$

Where,  $r$  is the interatomic separation,  $\phi'''(r)$  and  $\phi''(r)$  are the third and second order derivatives of potential energy. The second Gruneisen parameter is given as,

$$q = \gamma [1 + 2\gamma\alpha_v T] \quad (10)$$

The ratio of volume expansion coefficient  $\alpha_v$  to the volume specific heat  $c_v$  is calculated from its well known expression,

$$\frac{\alpha_v}{c_v} = -\left[ \frac{\phi'''(r)}{2r\phi''(r)} \right]_{r=r_0} \quad (11)$$

The Moelwyn Hughes constant ( $F_1$ ) has been calculated from the relation,

$$F_1 = 1 - \frac{\beta}{27K} [\phi'''(r)]_{r=r_0} \quad (12)$$

**b. Elastic properties:** We have studied the anharmonic elastic properties by calculating Second Order Elastic Constants (SOECs), Third Order Elastic Constants (TOECs) as they provide physical insight into the nature of binding forces between the constituents of the crystal and are strongly related to other anharmonic properties, like thermal expansion, thermoelastic constants and thermal conductivity. The expression for these elastic constants and the pressure and temperature derivatives of SOECs are given by [31-35]

$$c_{111} = 32.237Tb\beta_T + \frac{e^2}{4r_0^4} [-32.231B_1 - 34.48B_2 + \frac{4C_1 - 12A_1 + C_2 - 3A_2}{4} + 13.4599B_1 + 89.305r_0 f'(r_0)] \quad (13)$$

$$c_{112} = -4.15Tb\beta_T + \frac{e^2}{4r_0^4} [4.151B_1 + 4.52B_2 + \frac{C_2 - 3A_2}{8} + 4.6608r_0^2 f''(r_0) - 18.640r_0 f'(r_0)] \quad (14)$$

$$c_{166} = -6.151Tb\beta_T + \frac{e^2}{4r_0^4} [4.151B_1 + 4.52B_2 + \frac{C_2 - 3A_2}{8} - 3.5377B_{21} - 5.56r_0 f'(r_0)] \quad (15)$$

$$c_{123} = 2.332Tb\beta_T + \frac{e^2}{4r_0^4} [-2.332(B_1 + B_2) + 16.692r_0 f'(r_0)] \quad (16)$$

$$c_{144} = 2.332Tb\beta_T + \frac{e^2}{4r_0^4} [-2.332(B_1 + B_2) + 5.564r_0 f'(r_0)] \quad (17)$$

$$c_{444} = 2.332Tb\beta_T + \frac{e^2}{4r_0^4} [-2.332(B_1 + B_2)] \quad (18)$$

$$\text{where } T\beta_T = \frac{e^2}{4r^4} [1.165\varepsilon\{\varepsilon + 12f(r)\} + B_1 + B_2]$$

and  $A_1, A_2, B_1, B_2$  are the short range force parameters;  $f(r)$  and  $rf'(r)$  are the TBI parameters. The parameter  $f(r)$  and its derivatives are calculated from the knowledge of elastic constants and the  $f = f_0 \exp(-r/\rho)$ .

The expressions for the pressure derivatives of SOECs and TOECs are evaluated by using the expressions of second order, third order and fourth order elastic constants

$$\frac{dK}{dp} = -(3\Omega)^{-1} \left[ \frac{2.0350\varepsilon Z_m^2 + C_1 - 3A_1 + C_2 + 3A_2}{-56.4032\varepsilon r_0 f'(r_0) + 24.433\varepsilon r_0^2 f''(r_0)} \right] \quad (19)$$

$$\frac{dS}{dp} = -(2\Omega)^{-1} \left[ \frac{-5.2235\varepsilon Z_m^2 + A_1 - B_1 + \frac{C_2}{2}}{+47.954\varepsilon r_0 f'(r_0)} \right] \quad (20)$$

$$\frac{dc_{44}}{dp} = -(\Omega)^{-1} \left[ \frac{1.2323\varepsilon Z_m^2 + \frac{A_1 - 7B_1 + C_1 + 3A_2 - 9B_2}{6}}{-6.580\varepsilon r_0 f'(r_0) + 2.714\varepsilon r_0^2 f''(r_0)} \right] \quad (21)$$

$$\Omega = -0.3392\varepsilon Z_m^2 + \frac{A_1 + A_2}{2} + 9.4008\varepsilon r_0 f'(r_0)$$

$$S = \frac{c_{11} - c_{12}}{2}, \quad K = \frac{c_{11} - c_{12}}{3}$$

In addition to the pressure derivatives we have also evaluated the temperature derivatives of second order elastic constants [7] as given below

$$\frac{dc_{11}}{dT} = [c_{111} + 2c_{112} + 2c_{11} + 2c_{12}] \alpha_V \quad (22)$$

$$\frac{dc_{12}}{dT} = [c_{123} + 2c_{112} + c_{12}] \alpha_V \quad (23)$$

$$\frac{dc_{44}}{dT} = [c_{144} + 2c_{144} + 3c_{12} + 3c_{44}] \alpha_V \quad (24)$$

Where,  $\alpha_V$  is the volume thermal expansion coefficient. Using these equations, we have computed the temperature derivatives of second order elastic constants for  $C_{60}$

### Input data and model parameters

Using the input data given in Table 1 for fcc  $C_{60}$  [36-37] we have calculated the model parameters listed in Table 2.

Table 1. Input data at 300K for  $C_{60}$

S.No.	Properties	$C_{60}$
1	r (Å)	3.55 <sup>a</sup>
2	$c_{11}$ ( $10^{11}$ dyn/cm <sup>2</sup> )	2.64 <sup>a</sup>
3	$c_{12}$ ( $10^{11}$ dyn/cm <sup>2</sup> )	1.38 <sup>a</sup>
4	$c_{44}$ ( $10^{11}$ dyn/cm <sup>2</sup> )	1.17 <sup>a</sup>

a-[36]

Table 2. Model parameters at 300K for  $C_{60}$

S.No.	Model Parameters	$C_{60}$
1	b ( $10^{12}$ erg)	4.164
2	$\rho$ (Å)	0.2280
3	f (r)	-0.0041

### Results and discussion

Results and discussion are given below in the next sections:

**a. Lattice static properties:** We have calculated the compressibility ( $\beta$ ), molecular force constant ( $f$ ), Reststrahlen frequency ( $\nu_0$ ), Debye temperature ( $\Theta_D$ ), Gruneisen parameter ( $\gamma$ ), second Gruneisen parameter ( $q$ ), Moelwyn Hughes constants ( $F_i$ ), ratio of volume expansion coefficient ( $\alpha_V$ ) to volume specific heat ( $c_V$ ), which are directly derived from the cohesive energy  $\phi(r)$ . The Table 2 shows that our values on cohesive energy and compressibility ( $\beta$ ) are in good agreement with their available data [37]. The other thermodynamic properties listed in table 3 could not be compared due to lack of experimental data.

Table 3. Cohesive and Thermal properties of  $C_{60}$  at 300K

Properties	Values
$\phi$ (eV)	-1.096(-1.99 <sup>b</sup> )
$\beta$ ( $10^{-12}$ dyn.cm <sup>-1</sup> )	1.669
$f$ ( $10^4$ dyn.cm <sup>-1</sup> )	1.267
$\nu_0$ (THz)	3.807
$\Theta_D$ (K)	182.76

$\alpha_V/c_V(10^3J)$	5.529
$\gamma$	2.899
Q	2.958
$F_1$	6.714

b-[37]

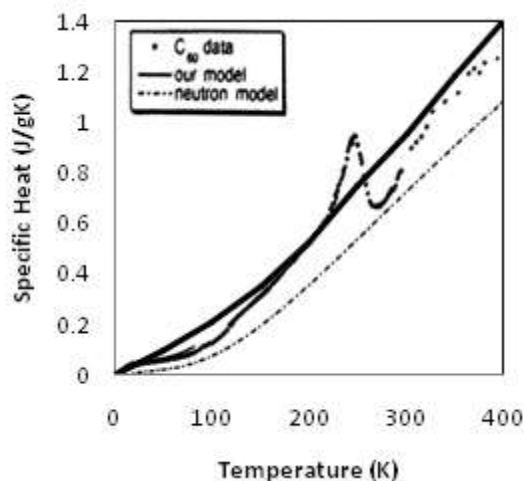
**b. Anharmonic elastic properties:** We have studied the anharmonic elastic properties by calculating Second Order Elastic Constants (SOECs), Third Order Elastic Constants (TOECs). We have calculated the TOECs for  $C_{60}$  probably for the first time at 300K are listed in Table 4. Our results on TOECs are found in good agreement with the available data[38]

*Table 4. Values of pressure derivatives of SOECs and TOECs ( $10^{12}$  dyn/cm<sup>2</sup>) of  $C_{60}$  at 300K*

Properties	Values	Properties	Values
$c_{111}$	-7.997( <b>6.240</b> ) <sup>c</sup>	dK/dP	16.14
$c_{112}$	-3.830( <b>-1.905</b> ) <sup>c</sup>	dS/dP	2.977
$c_{166}$	-1.800( <b>-1.605</b> ) <sup>c</sup>	dc <sub>44</sub> /dP	-0.555
$c_{123}$	1.272 ( <b>-1.80</b> ) <sup>c</sup>	dc <sub>11</sub> /dT	-5.176
$c_{166}$	1.021	dc <sub>12</sub> /dT	-0.384
$c_{456}$	0.895	dc <sub>44</sub> /dT	-0.092

a – [38]

**c. Specific heat curve:** We have reported the specific heat variation of  $C_{60}$  within the temperature range  $0K \leq T \leq 400K$  and shown in Fig. 1 along with experimental results [39, 40]. Our computed specific heat as a function of temperature follow the similar trend as experimentally observed curve. The specific heat in the normal state of the material is usually approximated by the contribution of the lattice and electronic specific heat. In our ETSM model calculation, we have considered only the lattice contribution, which causes the discrepancy from measured specific heat curve. On the basis of above discussions, it may be concluded that ETSM has successfully explained the cohesive and thermal properties of  $C_{60}$ . The result on specific heat can further be improved by incorporating the electronic contribution in specific heat expression.



*Figure 1: Variation of Specific heat of  $C_{60}$  in the temperature ranges  $0K \leq T \leq 400K$ . The solid line (—) are calculated and compared with microcalorimeter (■) and neutron model(- -) results [39, 40].*

## Conclusion

The present ETSM has been applied, probably for the first time, to explore the elastic, cohesive and thermal properties of the orientationally disordered  $C_{60}$ . We have also applied our model for the calculation of specific heat curve. It can be concluded on the basis of the above result and discussions that the agreement between the experimental [37-40] and our theoretical results for elastic, cohesive and thermal properties are reasonably good for some of the parameters and some of them could not be compared due to lack of experimental data. A detailed description of the theoretical investigation of the second and higher order elastic, cohesive and thermodynamic properties of the orientationally disordered  $C_{60}$  has been presented in this paper. Here, some of the results on the higher order elastic constants and thermophysical parameters are of academic interest at present but they may serve as a guide to the exper- mental workers in future.

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